

Experimental Test of Relation between Coherence and Path Information

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Quantum coherence defined by the superposition behavior of a particle beyond the classical realm, serves as one of the most fundamental features in quantum mechanics. Meanwhile, the wave-particle duality phenomenon, which shares the same origin, therefore has a strong relationship with the quantum coherence. Recently an elegant relation between the quantum coherence and the path information has been theoretically derived [Phys. Rev. Lett. 116, 160406 (2016)]. Here, we demonstrate an experimental test of such new duality by l_1 -norm measure and the minimum-error state discrimination. We prepare three classes of two-photon state encoded in polarization degree of freedom, with one photon served as the target and another photon as the detector. We observe clear wave-particle like complementarity by varying detector states in different region of inherently embodied quantity of quantum coherence. Our results may shed new light on original nature of wave-particle duality and applications of quantum coherence as a fundamental resource in quantum technologies.

Introduction - Coherence was early recognized as superposition of optical field in the theory of electromagnetic waves. In a combination with energy quantization, quantum version of coherence has become one of most fundamental features that can mark the departure of quantum mechanics from the classical realm [1–4]. Executing general quantum operations remotely under local operations and classical communication requires quantum states that contain a relevant resource and can be consumed. It was found that quantum entanglement as a typical resource has strong connection with coherence [5–7] and even may origin from it [8]. The development of quantum technologies demands a reassessment of quantum coherence as such fundamental resource, including relation with other quantum physical phenomena and rigorous quantitative description [9–15].

Among these quantum physical phenomena, wave-particle duality has been a unified picture to fully describe the behavior of quantum-scale objects. Quantitative characterization of such wave-particle duality relations have been extensively investigated, aiming to set an upper bound, the sum of the wave behavior and the particle behavior for a given interferometer [3, 4]. Based on the fringe visibility V of the interference pattern and the path distinguishability D , a wave-particle duality relation is given by $V^2 + D^2 \leq 1$, which means that full wave-behavior ($V = 1$) implies no particle behavior ($D = 0$) and vice-versa.

Recently, Emilio Bagan et.al [16] proposed another elegant relation to characterize the wave-particle duality based on the coherence C^{l_1} which quantifies the wave nature of a state and on the path information that is given by the minimum-error state discrimination between the detector state and the target state. In this paper, we experimentally test such new duality relation by l_1 -norm measure and the minimum-error state discrimination. By preparing three classes of two-photon state that have different inherently embodied quantity of quantum coherence, we are able to investigate wave-particle-like complementarity in different region. In every class of state, we continuously tune the detector state of one photon and observe clear duality tradeoff and the upper bound on the target photon.

Consider a particle enters an N-port interferometer via a generalized beam splitter. After the particle interacted with the detector, the state of the entire system is described by

$$|\psi\rangle = \sum_{i=1}^2 \sqrt{p_i} |i\rangle |\eta_i\rangle, \quad (1)$$

which is the superposition of the initial target particle state $|i\rangle$ and the detector state $|\eta_i\rangle$. The coherence of the particle state is given by $C = C^{l_1}(\rho)/N$, where $\rho = Tr_{det}(|\psi\rangle\langle\psi|) = \sum_{i,j=1}^N \sqrt{p_i p_j} \langle\eta_j|\eta_i\rangle |i\rangle\langle j|$. $C^{l_1}(\rho) = \sum_{i \neq j} |\rho_{ij}|$, where $|\rho_{ij}|$ is the absolute value of entry ρ_{ij} of the density matrix ρ under the reference basis[9]. Here, C quantifies the wave nature of the target particle.

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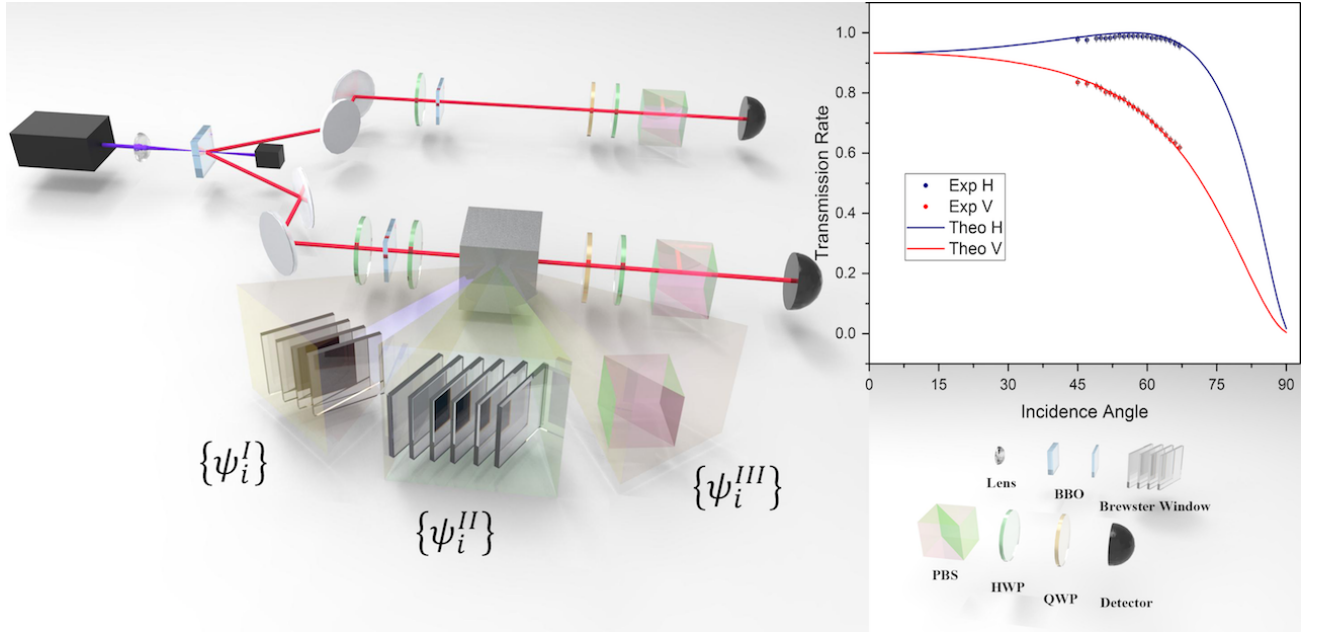


FIG. 1. **Experimental setup and transmission rate characterization of fused silica with different incidence angles.** Pairs of polarization entangled photons are generated via spontaneous parametric downconversion in a 2mm thick BBO crystal pumped by a 405nm UV laser. A combination of a half-wave plate (HWP) and a BBO of 1mm thick can be used as the compensation crystal. After rotating the polarization of one path to a superposition of horizontal and vertical polarization, we guide the photons to pass through the state generation section where three different classes of states $\{\psi_i^I\}$, $\{\psi_i^{II}\}$ and $\{\psi_i^{III}\}$ are generated. The states are then either analyzed by quantum state tomography through a section composed of a half-wave plate (HWP), a quarter-wave plate (QWP) and a polarization beam splitter (PBS) or by optimal minimum error measurement. The inset shows the experimental characterization of the Brewster window.

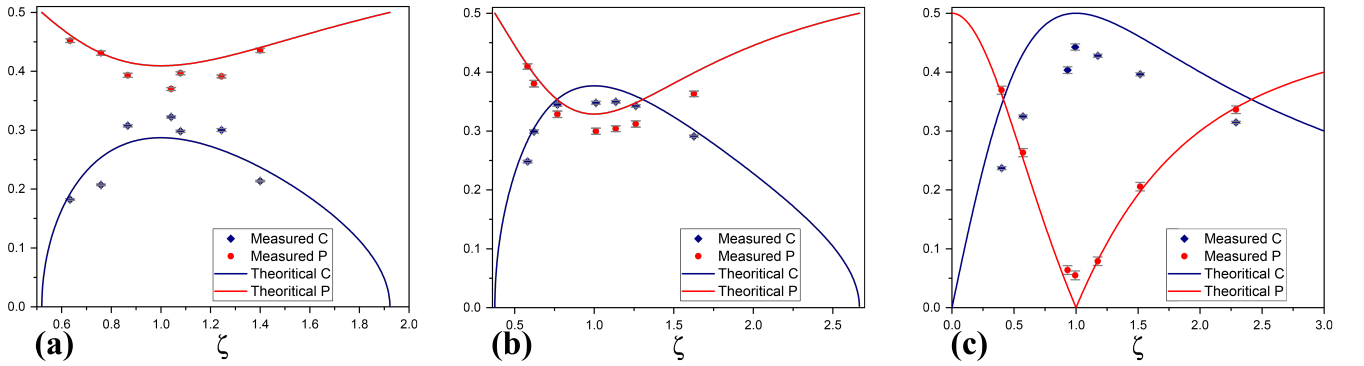


FIG. 2. **Experimental result of wave-particle duality in three different scenarios.** From (a) to (c), we experimentally test this new relation with three different classes of states we prepare ($\{\psi_i^I\}$, $\{\psi_i^{II}\}$ and $\{\psi_i^{III}\}$). Both the wave and particle properties are obtained from tomographic results. The blue points represent the coherence properties while the red ones give the path information. As we vary the detector states from $\{\psi_i^I\}$ to $\{\psi_i^{III}\}$, a continuous transition between the coherence and the path information can be observed while the degree of complementarity increases. The mismatch between the measured results and the theoretical predictions can be attributed to the imperfection of state preparation.

The detector state is given by tracing out the target particle state, $\rho_{det} = Tr_{tar}(|\psi\rangle\langle\psi|) = \sum_{i=1}^N p_i \rho_i$, where $\rho_i = |\eta_i\rangle\langle\eta_j|$. The detector states are introduced to quantify the path information of the target particle. If the detector states ρ_i are orthogonal, no wave property of the target particle will be obtained. To discriminate among the detector states $|\eta_i\rangle$, one employs the minimum-error strategy by using an N -element positive operator val-

ued measure (POVM) with elements $\{\Pi_i\}$. Then the average probability of successfully identifying the state is $P_s = \sum_{i=1}^N p_i \langle\eta_j|\Pi_i|\eta_i\rangle$. It is proved that C and P_s satisfy the following this new relation raised by Ref.[16],

$$(P_s - \frac{1}{N})^2 + C^2 \leq (1 - \frac{1}{N})^2. \quad (2)$$

This upper bound represents a trade-off between the par-

ticle and wave nature of the target particle. In our experiment, we consider the case of $N = 2$. In this case, P_s can be maximized either by an optimized POVM or calculated by an analytic solution.

We first experimentally generate a two-photon polarization entanglement state via type-II spontaneous parametric down-conversion[17]. As shown in Fig.1, a 405nm UV laser is focused on a 2mm type-II degenerate non-collinear cut BBO (beta-barium-borate) to generate a polarization-entangled singlet state $|\psi^-\rangle = (|H_1V_2\rangle - |V_1H_2\rangle)/\sqrt{2}$. Birefringence-induced spatial and temporal distinguishability can be compensated by a combination of a half-wave plate (HWP) and a BBO of 1mm thick).

In order to observe the coherence properties of the target, the detector states should be non-orthogonal. In our experiment, we use a HWP to generate a four-component state, which can be written as

$$|\psi\rangle = \alpha|H_1H_2\rangle + \beta|H_1V_2\rangle + \gamma|V_1H_2\rangle + \delta|V_1V_2\rangle \quad (3)$$

α , β , γ and δ are parameters based on the rotation in Hilbert Space. Since $|\psi^-\rangle$ state is invariant under arbitrary unitary rotation in Hilbert space, polarization-

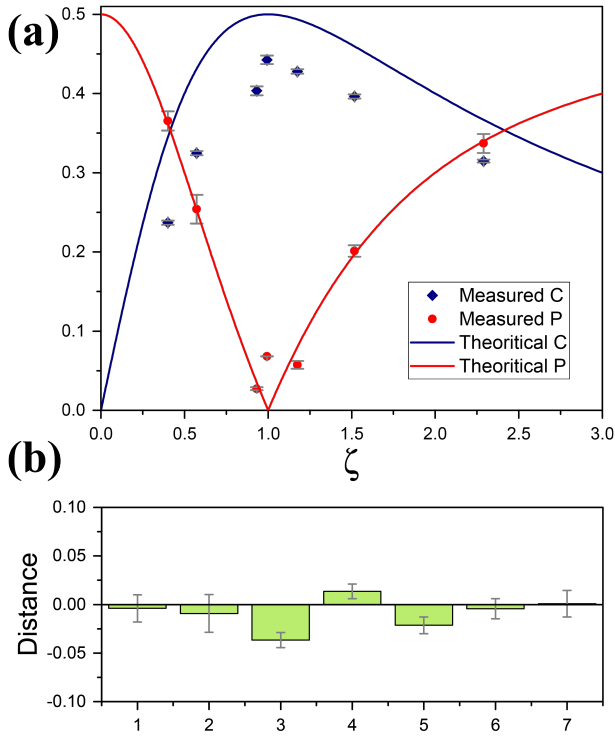


FIG. 3. (a) Coherence of l_1 -norm measure and path information given by direct optimized POVM results of the third class state $\{\psi_i^{III}\}$. (b) P_s result comparison between the analytic solutions and optimized POVM. Complementarity results of states $\{\psi_i^{III}\}$ between coherence C and path information P obtained by optimized POVM. Differences between the two kinds of result (direct measurement and analytic solution) are calculated and the results agree well with each other, less than 0.05.

dependent loss should be additionally introduced to generate non-orthogonal basis of the detector states. Here, we choose fused silica as Brewster window to introduce this polarization-dependent loss into the above four components.

We experimentally characterize the transmission rate of fused silica with different incidence angles and the results agree well with the theoretical prediction (see inset in Fig.1). We choose 60° as the incidence angle where the

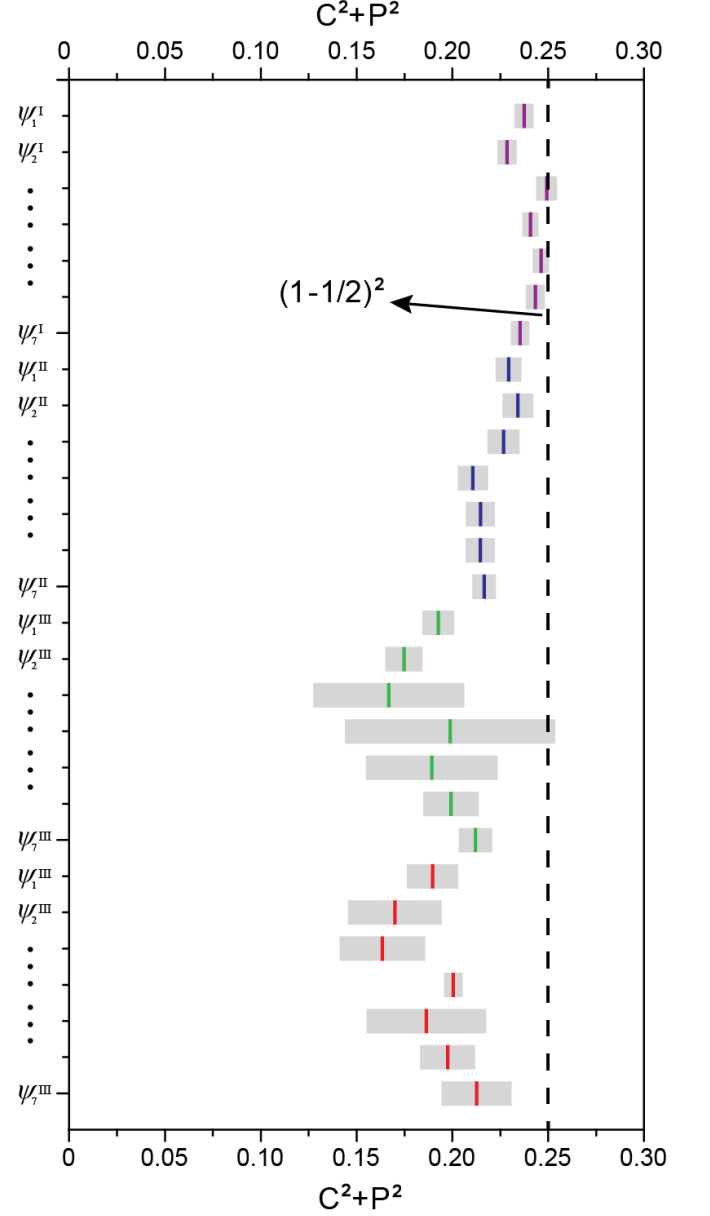


FIG. 4. The upper bound defined by the new wave-particle duality relation and experimental results from different states as input states. Sum of C and P square of different input states compared with theoretical upper bound. For the third class states $\{\psi_i^{III}\}$, both direct measurement and tomographic results are listed. All the states obey this new wave-particle duality relation.

transmission rate is 99.7% and 71.9% for horizontal and vertical polarization respectively (ϵ_h and ϵ_v). Then the two photons pass through this state preparation section and through recombination the states can be described as

$$|\psi\rangle = |H_1\rangle(\alpha\epsilon_h|H_2\rangle + \beta\epsilon_v|V_2\rangle) + |V_1\rangle(\gamma\epsilon_h|H_2\rangle + \delta\epsilon_v|V_2\rangle) \quad (4)$$

We define parameter ζ as

$$\zeta = \sqrt{\frac{\alpha^2\epsilon_h^2 + \beta^2\epsilon_v^2}{\gamma^2\epsilon_h^2 + \delta^2\epsilon_v^2}} \quad (5)$$

By tuning the angles of the half-wave plate (affect α , β , γ and δ) or the pieces of Brewster window (affect ϵ_h and ϵ_v), we are able to generate different classes of states which we donate as $\{\psi_i^I\}$ and $\{\psi_i^{II}\}$ to test this new duality relation. Note that the two-photon state is still polarization entangled. As we vary among different classes of states, the concurrence of the entangled state decreases, from 0.7007 to 0.5664. For an asymptotic limit case, we use a polarization beam splitter (PBS) to replace the Brewster window to generate a separable state ($\{\psi_i^{III}\}$).

These output states are then analyzed by quantum state tomography with maximum likelihood reconstruction[18]. From the tomographic details, we are able to calculate both C and $P = P_s - \frac{1}{N}$ to estimate the wave and particle nature of the target. For different classes of states, we measure this coherence-path information duality complementarity in three cases and the results are shown in Fig.2. For all the three cases, while we tune the parameter ζ , we are able to observe continuous transitions between the coherence and the path information. The distances between the experimental results and the theoretical values are caused by the imperfection preparation of the entanglement state. Especially for the third case, the maximum coherence C^{I_1} measure is limited by the polarization visibility of the entanglement state.

Apart from direct calculation of the successful discrimination probability from the tomographic data, we also use optimized POVM to directly measure the maximum probability of successfully identifying the state in the third scenario where maximum complementarity can be obtained. The results are given in Fig.3 as well as the

mismatch between direct measurement and analytic solution. The distances between the two different results are less than 0.05 which proves that the two methods are equivalent in principle.

Finally, we test the upper bound of this new duality relation by adding the square of both C and P of all three classes of states we prepared. For $\{\psi_i^{III}\}$, two different results are given based on direct measurement and analytic solution. The results are shown in Fig.4. All C and P errors are estimated via 1,000 rounds Monte Carlo simulation based on photon number detection with Positionian fluctuation. Then the errors of the final value are given by the error propagation theory. Large error bars come from the uncertainty when we measure rather small observables. All data obey this new duality relation with tolerable errors and this result is implied by the Bohrs principle of complementarity.

The wave-particle duality relations are of significance in quantum mechanics. And the quantum coherence plays central roles in quantum information processing. The quantum coherence based wave-particle duality relation presented in Ref[16] gives a first inequality satisfied by the coherence and the path information given by the minimum-error state discrimination between the detector state and the target state. By using two-photon states encoded in polarization degree of freedom, one photon for the target and another photon for the detector, our experiment has demonstrated this coherence based wave-particle duality relation for various detector states. Our results may highlight further theoretical and experimental investigations on such fundamental researches and strengthen the prominent role of coherence in quantum physics and quantum technologies.

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